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# Implementation of Segmented Circular-Arc Constant Beamwidth Transducer (CBT) Loudspeaker Arrays

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## ABSTRACT

Circular-arc loudspeaker line arrays composed of multiple loudspeaker sources are used very frequently in loudspeaker applications to provide uniform vertical coverage [1, 2, and 4]. To simplify these arrays, the arrays may be formed using multiple straight-line segments or individual straight-line arrays. This approximation has errors because some of the speakers are now no longer located on the circular arc and exhibit a “bulge error”. This error decreases as the number of segments increase or the splay angle of an individual straight segment is decreased.

The question is: How small does the segment splay angle have to be so that the overall performance is not compromised compared to the non-segmented version of the array? Based on two simple spacing limitations that govern the upper operating frequency for each type of array, this paper shows that the bulge deviation should be no more than about one-fourth the center-to-center spacing of the sources located on each straight segment and that surprisingly, the maximum splay angle and array radius depends only on the number (N) of equally-spaced sources that are on a straight segment. As the number of sources on a segment increases, the maximum segment splay angle decreases and the required minimum array radius of curvature increases. Design guidelines are presented that allow the segmented array to have nearly the same performance as the accurate circular arc array.

## 1. INTRODUCTION

Circular-arc loudspeaker line arrays are used very frequently in loudspeaker applications to provide uniform vertical coverage. These arrays may be approximated by forming the array using multiple straight-line segments where each segment is a straight-line array which contains multiple drivers. This approximation has errors because each speaker is now no longer located on the circular arc.

A bulge error can be defined which describes the maximum deviation of a source on the straight line segment from the corresponding point on the circular arc. The bulge error decreases as the number of segments increase.

The question is how many segments are required so that the overall performance is not compromised compared to the non-segmented version of the array?

Analysis reveals two spacing conditions that roughly govern the upper operating frequency of each type of array: 1) the un-segmented circular arc array operates well up to the frequency where the center-to-center source spacing is about one-half wavelength, and 2) compared to the pure circular-arc array, the segmented array operates well up to the frequency where the bulge is about one-eighth wavelength.

In the first case the un-segmented circular arc array exhibits a chaotic loss of high-frequency control and the appearance of detrimental side lobes and grating lobes [3] at higher frequencies.

In the second case the HF limitation is related to how far an individual source is away from the circular arc.

Forming the ratio of these two conditions indicates that the bulge should be no more than about one-fourth the center-to-center (C-C) spacing of the sources. This condition places constraints on the splay angle of the straight-line segments in the circular-arc array and on the array's overall radius of curvature.

Surprisingly, analysis shows the maximum splay angle and the array's minimum radius of curvature depends only on the number (N) of equally-spaced sources that are on a straight-line segment. As the number of sources on a segment increases, the maximum segment splay angle decreases and the minimum array radius increases.

Example: the maximum splay angle for four speakers on segment is about 30 degs but for 16 speakers the maximum splay angle is only about 8 degs. The corresponding array radius increases from 2L to 4L where L is the segment length.

## 2. SEGMENTED CIRCULAR ARCS

Multiple small straight arrays can't directly implement a true circular-arc array. They can however, implement an approximation to a circular arc called a "segmented" circular arc (https://en.wikipedia.org/wiki/Circular_segment). Fig. 1 shows an approximation to a 90° arc with respectively three, four, and six segments.

Notice that the "Bulge", or the maximum deviation of the straight line segment from the circular arc, decreases considerably as the number of segments increase. The six segment approximation is nearly indistinguishable from the true circular arc! At the end of this paper is an appendix that derives formulas for these relationships.

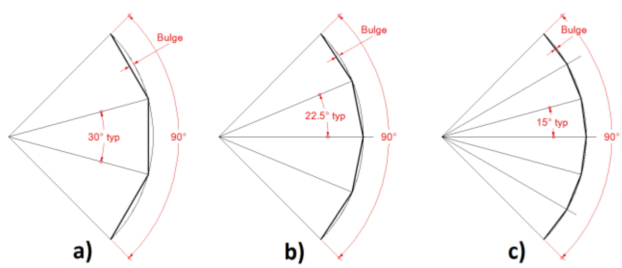


**Fig. 1. Segmented approximations to a 90° circular arc. a) Three segments, b) Four segments, c) Six segments.**

## 3. SEGMENTED CIRCULAR-ARC LOUDSPEAKER ARRAYS

In this section, the relationships between multiple sources on a circular arc and the same sources arranged on several straight segments that approximate the circular arc are investigated.

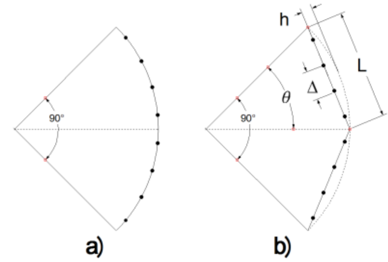


**Fig. 2. a) A pure eight-source 90° circular-arc array. b) A two-segment approximation of the same array with four sources per segment. The bulge error is *h*.**

Fig. 2 illustrates two versions of an eight-source 90° loudspeaker array. The left image is the desired pure circular arc version. The right image shows the same array approximated with two straight-line four-source segments.

Essentially the second version implements the array with two four-source straight arrays. The assumption here is that two four-source straight-line arrays are easier to implement and manufacturer than a single curved-front array.

The major concern here is how much the segmented approximation to the array compromises the performance of the pure circular-arc array.

Several questions come to mind: 1. How much bulge is acceptable (ratio between Δ and *h*, defined as $\beta$ here)? 2. What is the minimum number of segments required or alternately what is the maximum segment (or splay) angle θ for a specific number of sources on a segment? 3. Are there limitations on the array radius of curvature that depend on the specific number of sources on a segment?

## 4. HOW MUCH BULGE IS ACCEPTABLE?

Analysis shows (not shown) that the segmented circular-arc array operates well up to the frequency $F_{max}$ where the bulge $h$ is about one-eighth wavelength $(h_{\mathrm{m}} = \lambda/8)$. Above this frequency the sources are displaced to far away from the true circular arc with respect to wavelength and that their acoustic outputs do not sum accurately.

The upper operating frequency $F_{max}$ of a pure circular-arc CBT array is governed by the C-C spacing Δ of the discrete sources that makes up the array. The array controls beamwidth up to the frequency where Δ is about one-half wavelength. $(\Delta_{\mathrm{m}} = \lambda/2)$. Above this frequency, side lobes and grating lobes appear and the coverage goes chaotic. Operating the array in this chaotic range may not be a bad thing because the vertical coverage suddenly gets very broad and this may be judged a plus rather than a minus depending on the array's application.

Forming the ratio of these two conditions at $F_{max}$ indicates that the bulge should be no more than about one-fourth the C-

C spacing of the sources ($h_m/\Delta_m = 1/4$). This ratio is defined as $\beta$. This condition places constraints on the splay angle of the modules in the circular-arc array and on the array's radius of curvature. Refer to the next section and the appendix where these relationships are clarified and developed.

## 5. MAXIMUM SEGMENT ANGLE AND MINIMUM ARRAY RADIUS

Each straight array or straight segment that forms the total array has a maximum splay angle and minimum radius of curvature that when exceeded violates the rule pointed out in the previous section.

Surprisingly, the maximum splay angle and the array's radius of curvature depend only on the number (*N*) of equally-spaced sources that are lined up on the segment. This relationship is shown in the following table. The equations used to calculate the values in the table and their derivations are shown in the appendix (Eqs. 6 and 7). The bulge to C-C spacing ratio $\beta$ was set to $1/4^{th}$ for the values in this table. All numbers have been rounded off to nearest usable values.

**Table 1: Maximum Segment Angle and Minimum Array Radius vs. Number of Speakers on a Segment**

| Number of Speakers on Segment, N | Approximate Maximum Segment Angle or Splay Angle, θ | Approximate Minimum Array Radius of Curvature, R (Where L = Segment Length) |
|---|---|---|
| 2 | 56° | L |
| 4 | 28.5° | 2L |
| 8 | 14.3° | 4L |
| 16 | 7.2° | 8L |
| 32 | 3.6° | 16L |

Example: When the number of sources on a segment is eight the maximum segment angle is 14.3° or about 15° and the minimum array radius of curvature is about four segment lengths.

## 6. ARRAY SIMULATIONS

To illustrate the performances differences between a pure circular-arc array and its segmented approximations, I simulated three 48-source 1 m tall arrays. This could possibly be an array using 48 each very-small wide-band HF radiators.

Three 90° arrays were compared: 1) For reference, a pure CBT circular-arc array with 3 dB stepped and 12 dB truncated Legendre shading [4], 2) A three-segment version of this array with 16 sources per segment, and 3) A six-segment version of array of the same array with 8 sources per segment. Array two violates the guidelines recommended in this paper while array three meets the guidelines.

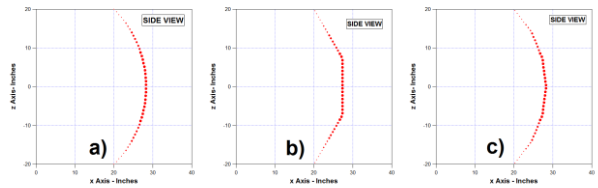


**Fig. 3. Simulated 90° one-meter tall 48-source arrays. a) Pure circular-arc CBT array. b) Three segment approximation. c) Six segment approximation. Dot size indicates shading amplitude.**

The following three subsections show selected simulation results that illustrate the performance of the three arrays. Simulation results include: 1. beamwidth (-6 dB) vs. frequency, 2. full-sphere directivity vs. frequency, and 3. vertical polar shape at 8 kHz. All simulations were accomplished using the Igor Pro simulator described in [1, 3.1 Introduction]. Note that the polar shapes are bidirectional due to the simulator's use of point sources.

### 6.1. Reference CBT Circular-Arc Array

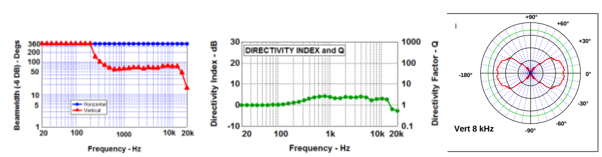


**Fig. 4. Beamwidth, directivity, and 8 kHz vertical polar pattern (left to right respectively) of the reference CBT array shown in Fig. 3a. Note the very uniform results. The polar pattern exhibits a bidirectional symmetrical right-left shape because the simulator uses omni-directional point sources.**

### 6.2. Three-Segment Array

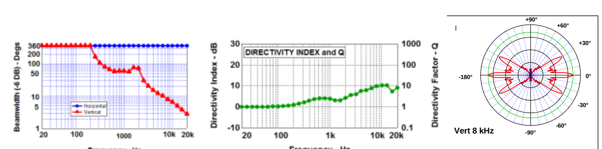


**Fig. 5. Beamwidth, directivity, and 8 kHz vertical polar pattern (left to right respectively) of the three-segment array shown in Fig. 3b. Note the very-poor results and distorted polar shape.**

### 6.3. Six-Segment Array

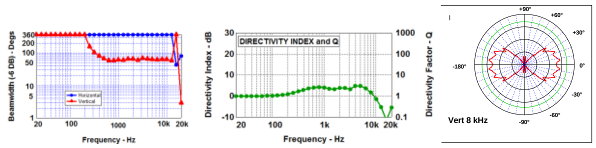


**Fig. 6. Beamwidth, directivity, and 8 kHz vertical polar pattern (left to right respectively) of the six-segment array of Fig. 3c. Note uniform results when compared to Fig. 4.**

### 6.4. Simulation Observations

The results of the reference CBT array (Fig. 4) exhibit very uniform beamwidth, directivity, and polars nearly independent of frequency up to above 10 kHz. The three-segment array results (Fig. 5), which violates the guidelines in this paper, exhibits very-poor performance with fair results only up to about 1.6 kHz and has a very distorted polar pattern. In contrast, the six segment array results (Fig. 6), which does follow the guidelines in this paper, exhibits decent results up to above 8 kHz.

## 7. SUMMARY

This paper has provided guidelines for creating segmented circular-arc loudspeaker arrays that minimize the performance limitations suffered when the array is converted to segments.

The paper pointed out that the segment's maximum splay angle and the array's minimum radius of curvature depends only on the number (N) of equally-spaced sources that are on a straight-line segment. As the number of sources on a segment increases, the maximum segment splay angle decreases and the minimum array radius increases.

A table was presented that provides design guidance for selection of segment splay angle and array radius of curvature given the number of sources on a segment.

## 8. REFERENCES

[1] D. B. Keele, Jr., "The Application of Broadband Constant Beamwidth Transducer (CBT) Theory to Loudspeaker Arrays," 109th Convention of the Audio Engineering Society, Convention paper 5216 (Sept. 2000).

[2] Mark S. Ureda, "Analysis of Loudspeaker Line Arrays", J. Aud. Eng. Soc., vol. 52, no. 5, pp. 467-495 (May 2004).

[3] Online references: http://www.olympus-ims.com/en/ndt-tutorials/transducers/lobes/. See also http://en.wikipedia.org/wiki/Side_lobe.

[4] D. B. Keele, Jr., "Practical Implementation of Constant Beamwidth Transducer (CBT) Loudspeaker Circular-Arc Line Arrays," presented at the 115th Convention of the Audio Engineering Society, New York, Convention paper 5863 (Oct. 2003).

## 9. APPENDIX: EQUATION DEVELOPMENT

### 9.1. Development of Circle Segment Formulas

This section draws heavily on the "Dr. Math" online forum:http://mathforum.org/dr.math/faq/faq.circle.segment.html.

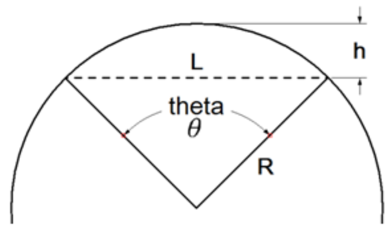


**Fig. 7. Diagram of a circle segment, bounded by an arc of the circle and the chord *L* subtending it.**

Let the length of the chord or segment length be *L*, the radius of the circle be R, the distance from the midpoint of the chord to the midpoint of the arc be *h* (or the bulge), and the measure in radians of the central angle subtending the arc be theta (where $0 \le \theta \le \pi$).

The radius *R* and segment angle $\theta$ can be shown to be:

$$R = (L^2 + 4h^2)/8h \qquad [1]$$

and

$$\theta = 2\sin^{-1}(L/2R). \qquad [2].$$

### 9.2. Number of Sources on a Segment

For this paper, several acoustic sources are equally spaced along the chord or segment *L* with C-C spacing $\Delta = L/N$ where *N* is the number of sources on the segment. The end spacing is half this value, i.e. $\Delta/2$. Fig.8 shows an example of three sources on a segment.

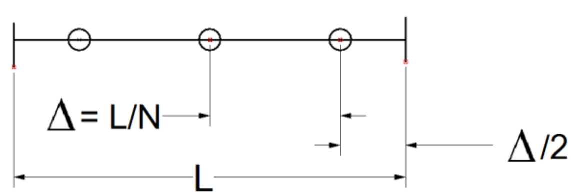


**Fig. 8. Three equally spaced sources on segment.**

### 9.3. Relate Circle Radius and Segment Angle to Segment Length and Bulge

Now relate the bulge *h* to the source spacing $\Delta$ by defining a parameter $\beta$ which is a multiplier that indicates how high the bulge is in terms of the source spacing.

$$h = \beta\Delta = \beta L/N \qquad [3]$$

Usually $\beta$ is much less than 1 and in the range of 1/8th to 1/4th. i.e. the bulge is 1/4th or less than the source spacing.

The value of bulge $h$ in terms of in terms of $\beta$ and $N$ (Eq. 3] can be substituted in Eq. 1 to yield the circle radius in these new terms

$$R = L\left(\frac{N^2+4\beta^2}{8\beta N}\right) \qquad [4]$$

In turn, Eq. 2 which gives the segment angle along with Eq. 1 appears as

$$\theta = 2\sin^{-1}\left(\frac{4\beta N}{N^2+4\beta^2}\right) \qquad [5]$$

Typically a $\beta$ of 1/4th is quite workable, but 1/8th provides closer approximation to the performance of a true circular arc.

When a $\beta$ of ¼ is substituted in Eqs. 4 and 5 they appear as

$$R = L\left(\frac{N\hat{}2+1/4}{2N}\right) \qquad [6]$$

$$\theta = 2\sin^{-1}\left(\frac{N}{N^2+1/4}\right) \qquad [7]$$

Surprisingly Eqs. 6 and 7 indicate is that that the circle radius and segment angle are essentially only a function of the number of sources on the segment! Eqs. 6 and 7 were used to create the data shown in Table 1 in Section 5.